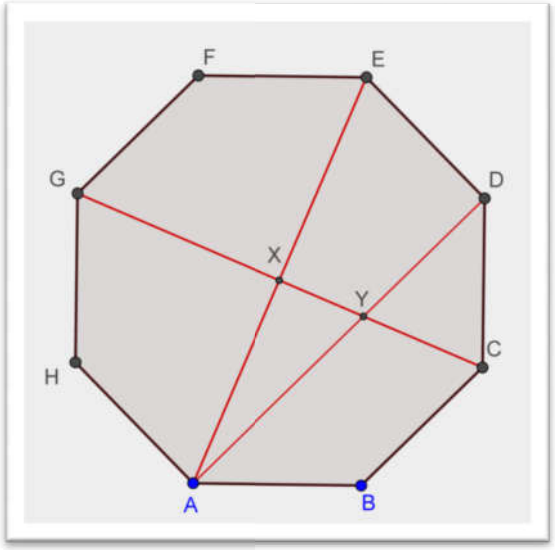
Polygons

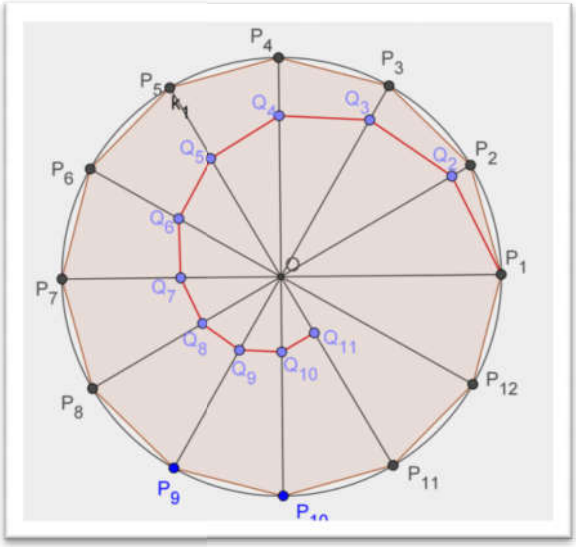
1. (Warm-up on Pythagoras Theorem)
Given a regular octagon ABCDEFGH with sides 2 cm.



(a) If AE intersects CG at X.
Find the length of AX.

(b) If AD cuts CG at Y.
Find the length of XY.

2. A dodecagon of is placed inside a circle of radius 1 cm, and the twelve dividing points are joined to the circle's centre, producing twelve rays. Starting from P_1 a segment is drawn perpendicular to the next ray OP_2 in the anti-clockwise sense; and from the foot of this perpendicular another perpendicular segment is drawn to the next ray, and so on forever.
Taking $Q_1 = P_1$.



(a) Find the limit of the sum of the lengths of these segments:

$$Q_1Q_2 + Q_2Q_3 + Q_3Q_4 + Q_4Q_5 + \cdots$$
$$= \sum_{k=1}^{\infty} Q_kQ_{k+1}$$

(b) Find the limit of the area of the triangles :

$$\Delta OQ_1Q_2 + \Delta OQ_2Q_3 + \Delta OQ_3Q_4 + \cdots$$
$$= \sum_{k=1}^{\infty} \Delta OQ_kQ_{k+1}.$$

(c) (For more able students)
Instead of starting with the circle divided into twelve equal parts, we now to divide it into n equal parts. Let $\angle Q_1OQ_2 = \alpha$.

(i) Find the sum of the lengths: $\sum_{k=1}^{\infty} Q_kQ_{k+1}$

(ii) Find the limit of the area of the triangles : $\sum_{k=1}^{\infty} \Delta OQ_kQ_{k+1}$